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Dimensionality and geometry effects on a quantum carnot engine efficiency

Efectos de dimensionalidad y geometría en la eficiencia del motor quantum carnot

Herrera Alcantar Hiram Kalid[1], Carvajal García José Carlos[1], Rosales Pérez Osvaldo[1], Villarreal-Sánchez Rubén Cesar[2], Iglesias-Vázquez Priscilla Elizabeth[1]

[1]Facultad de Ciencias, Universidad Autónoma de Baja California, Ensenada, Baja California, México
[2]Facultad de Ingeniería, Arquitectura y Diseño, Universidad Autónoma de Baja California, carretera transpeninsular Ensenada-Tijuana 3917, colonia Playitas, Ensenada, Baja California, México

Autor de correspondencia: Priscilla Elizabeth Iglesias Vázquez, Facultad de Ciencias, Universidad Autónoma de Baja California, Carretera Transpeninsular Ensenada-Tijuana 3917, Colonia Playitas, Ensenada, Baja California, México. E-mail: piglesias@uabc.edu.mx.

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Resumen. - *Calculamos la eficiencia de un ciclo de Carnot cuántico para una partícula confinada en dos pozos de potencial infinitos diferentes, un pozo de potencial cilíndrico de radio variable y un pozo de potencial bidimensional cuadrado con periodicidad en uno de sus lados. Encontramos que la eficiencia depende directamente de la dimensionalidad y la geometría del pozo que confina a la partícula.*

Palabras clave: Ciclo de Carnot; Motor térmico; Confinamiento cuántico.

Abstract. - *We calculate the efficiency of a quantum Carnot cycle for a particle confined in two different infinite potential wells, a cylindrical potential well of variable radius and a two-dimensional square potential well with a periodicity in one of it sides. We find that the efficiency depends directly on the dimensionality and geometry of the well that confined the particle.*

Keywords: Carnot cycle; Heat engine; Quantum confinement.

1. Introduction

A classical heat engine is a device that extracts energy QH from a high temperature heat source, it generates work W with an amount of this energy and the rest is release into a low temperature drain. The efficiency η of a heat engine is defined by $\eta = W/QH$. It is well known that the heat engine reaches the highest possible efficiency following Carnot cycle model [1]. This cycle consists in a gas confined by a cylinder with a movable piston. Although classical heat engines have been extensively studied, it is of interest to study the systems and processes that could increase their efficiency. In recent years, with the developments of nanotechnology and quantum information processing, the study of quantum systems began to attract more attention. Consequently, the

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Quantum Heat Engines (QHE) have been proposed theoretically [2-13]. In QHE, rather than having a gas confined in a cylinder with a movable piston, it is considered a single particle confined by a quantum potential well that walls play the role of the piston by moving in and out. Current studies on QHE have considered different type of potential wells, for example, a single particle confined by one (1D), two (2D) or three-dimensional (3D) infinite square potential well [3-5]. Based on this, in the present paper we further calculate the efficiency of a quantum Carnot cycle considering a single particle confined by two different types of potential wells, an infinite cylindrical potential well and an infinite 2D square potential well with periodicity, as to our best knowledge, these cases have not been considered. Comparison between these two cases enable us to extend our understanding about the dimensionality and geometry effects of quantum confinement on the efficiency of a QHE.

2. Methodology

We consider a particle of mass m, confined by two different types of quantum potential wells: an infinite cylindrical potential well (CPW) of radius r, in this case the particle is confined in the space inside the CPW. Also, we consider an infinite 2D square potential well (SPW) of length a that has periodicity every $2\pi R$ in the y direction, thus the space where the particle moves are now on a cylinder that has length a and circumference $2\pi R$.

2.1 *Schrödinger equation and energy eigenvalues*

We start from the time independent Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\,\psi\,, \qquad (1)$$

where $\hbar$ is the Planck constant, V represents the potential, ψ is the wave function, E are the eigen energies obtained by the expectation value of the Hamiltonian [14], and ∇^2 is the Laplace operator in the corresponding coordinates of each of the potential wells, i.e., in cylindrical coordinates for the CPW and in 2D Cartesian coordinates for the SPW. We shall use the symbol S to denote the length of the different types of potential wells, i.e., $S = r,\ a$. Once the energies E of each case is obtained (see Table 1), we calculated the force F exerted on the wall of the wells, which is defined as the negative derivative of the energy [3].

$$F = -\frac{dE(S)}{dS} \qquad (2)$$

where the length S may vary. From table 1, we can see that each energy level state Ek,l is inversely proportional to the length of the well, i.e., Ek,l decreases as the length of the of the well increases, and vice versa, in this sense we can imagine that the walls of the potential well can move like a piston in a classical thermodynamics system [3].

Table 1. Energies obtained in each potential well. Here $k,\ l$ = 1,2,3, … are quantum numbers and zkl is the kth cero of the Bessel function of order one.

Potential well	Energies
CPW	$E_{k,l} = \frac{\hbar^2}{2mr^2} z_{kl}^2$
SPW	$E_{k,l} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi k}{a}\right)^2 + \left(\frac{l}{R}\right)^2\right]$

2.2 *Quantum Carnot cycle*

The authors of Ref. 3 calculated the efficiency of a quantum Carnot cycle by using a single particle confined by a 1D infinite square potential well. Using the procedure described in Ref. 3, we further investigate the efficiency of a quantum Carnot cycle by considering different type of potential well that have not been reported. The quantum analogue of classical Carnot cycle consists of four processes described below and illustrated in figure 1.

1. Isothermal expansion. Starting at the ground state, which corresponds to the potential well of length $S1$, we expand isothermally this length up to $S2$ and excite the second energy state of the system. In this process, a force $F1$ is applied and an amount of energy QH is absorbed by the system.

ISSN: 2594-1925

2. Adiabatic expansion. The expansion continues adiabatically up to $S3$ applying a force $F2$ on the wall, and the system remains in the second energy state.
3. Isothermal compression. We compress isothermally the length of the well down to $S4$ until the system is back in the ground state. A force $F3$ is applied.
4. Adiabatic compression. The compression continues down to $S1$, applying a force $F4$ on the wall. During this process the system remains in the ground state.

The area of the closed loop in figure 1 represents the work W done in a single cycle of the quantum Carnot engine [3]. There is an associated force F to each of the four process, from these, we calculate the total work W done during a full cycle by evaluating the following integrals.

$$W = \int_{S_1}^{S_2} F_1\, dS + \int_{S_2}^{S_3} F_2\, dS + \int_{S_3}^{S_4} F_3\, dS + \int_{S_4}^{S_1} F_4\, dS. \tag{3}$$

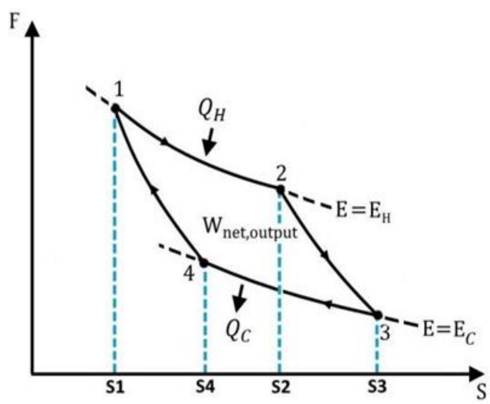

Figure 1. Illustration of a four-step quantum Carnot cycle, where ***S*** denote the length of the different types of potential wells and ***F*** is the force exerted on the wall of the wells.

We also calculate the energy QH absorbed by the system during the isothermal expansion, which is given by

$$Q_H = \int_{S_1}^{S_2} F_1\, dS\,. \tag{5}$$

Therefore, calculating W and QH for the CPW and SPW, we finally calculate the efficiency $\eta = W/QH$ of each case.

Table 2. Efficiency obtained in each potential well as a function of its length. Here $z_{11} \approx 3.8317$ and $z_{01} \approx 2.4048$.

Potential well	Efficiency
CPW	$\eta = 1 - \left(\frac{z_{11} r_1}{z_{01} r_3}\right)^2$
SPW	$\eta = 1 - 4\left(\frac{R_1}{R_3}\right)^2$

3. Results and Discussions

In the case of the SPW, we remained unchanged the non-periodical side on the x direction and the radius R of the periodicity on the other side was varied. For the CPW, the parameter that was varied was the radius of the cylinder. For each case, the procedure indicated in the methodology section was developed. The efficiencies obtained are shown in Tables 2 and 3.

Table 3. Efficiency of each potential well as a function of its energy level states

Potential well	Efficiency
SPW	$\eta = 1 - \left(\frac{E_C - \frac{\pi^2\hbar^2}{2ma^2}}{E_H - \frac{\pi^2\hbar^2}{2ma^2}}\right)$
CPW	$\eta = 1 - \frac{E_C}{E_H}$

4. Conclusions

It was found, from the efficiencies shown above, that the efficiency of the quantum Carnot cycle depends of the length of the potential well. It should be noted that the importance of this work relies in the fact that the efficiency changes as a function of the geometry and dimension of the potential well that confined the single particle, this could help for future works to find a QHE with a higher efficiency and possible applications such as those proposed in Ref. 11, 12 and 13, where possible applications are proposed for a

ISSN: 2594-1925

QHE. As a future work, other type of thermodynamic cycles such as the Otto cycle or Stirling cycle can be analyzed to determine how the dimensionality and geometry affects their efficiencies.

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