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Research article

Weibull strength distribution and reliability S-N percentiles for tensile tests

Análisis de resistencia Weibull para los percentiles S-N y su nivel de confiabilidad en test de tensión

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Abstract. - *Based on the true stress, the ultimate material's strength, and the fatigue slope b values, the probabilistic percentiles of the S-N curve of ductile materials are formulated. The Weibull β and η parameters used to determine the product's reliability are determined directly from the material's strength values corresponding to 103 and 106 cycles. And since in Table corresponding to the properties of this A538 A (b) steel and collected by table 23-A of Shigley Mechanical Engineering Design book; authors present the σt, Sut, and b values of several materials, then the Weibull parameters for each one of these materials as well as the 95% and 5% reliability percentiles of their S-N curves are given. A step-by-step application to the steel A538 A (b) material is presented. And based on the maximum and minimum applied stress values, the corresponding Weibull stress distribution was fitted and used with the Weibull strength distribution, in the stress/strength reliability function to determine the element's reliability.*

Keywords: Mechanical design; True stress-strain; Weibull distribution; Fatigue reliability analysis; Stress/Strength, Reliability Engineering.

Resumen. – *Basado en el estrés verdadero σ_t, la última resistencia del material S_ut, y la curva de fatiga b, la curva S-N de material de acero dúctil es formulada. La distribución Weibull con parámetros β y η son usados para determinar la confiabilidad del elemento y ambos son directamente determinados por la resistencia del material que en este caso corresponde a 103 y 106 ciclos. Y como corresponde en la tabla de propiedades del acero A538 A (b) y recolectada esta información del libro de Ingeniería mecánica de Shigley: los autores presentan el estrés verdadero, ultimo estrés y la curva de diferentes materiales. Entonces los parámetros Weibull β y η, así como los percentiles de confiabilidad 95 y 5 % de la curva S-N son presentados. Se presenta una aplicación paso por paso para el acero A538 A (b). Y basado en el máximo y mínimo estrés aplicado, la distribución Weibull correspondientes es presentada. Por último, basado en el máximo y mínimo estrés, la distribución Weibull correspondiente fue ajustada y usada con la resistencia de la distribución Weibull, en la función estrés-resistencia de confiabilidad con el objeto de estimar la confiabilidad del elemento.*

Palabras clave: Diseño mecánico; Estrés-resistencia; Distribución Weibull; Análisis de fatiga; Ingeniería de confiabilidad.

1. Introduction

Since the reliability of a mechanical component depends on the applied stress value and on the strength that the used material presents to overcome the applied stress, then because both the applied stress and the material's strength are random variables, then researchers have been proposing to use a probabilistic stress-cycles S-N curves. However, because the probabilistic percentiles of the S-N curves are based on the common confidence interval (*CL*) of the expected average, as shown in section 3.3, then the proposed formulations are inefficient to perform a reliability analysis.

Thus, in this paper based on the theory given in [1], a Weibull methodology to determine the *strength distribution* and *the reliability percentiles* of the S-N curve are both given. In the proposed Weibull/tensile test methodology, the only needed inputs are *1)* the ultimate material's strength [2] (S_{ut}) value, (which is a measure of the maximum stress that an object/material/structure can withstand without being elongated, stretched or pulled). *2)* the true stress (σ_t) [2] value, (which measures the change in the area with respect to the time while the specimen is loading), and *3)* the fatigue slope *b* value of the S-N curve. With these three inputs, the corresponding strength Weibull shape β and scale $\eta_{(\sigma)}$ parameters used to determine the reliability percentiles of the S-N curve, are both determined based on the $S_f = fS_{ut}$ strength value that corresponds to $N_1 = 10^3$ cycles and on the strength (S_e) value that corresponds to $N_1 = 10^6$ cycles. The validation that the addressed strength β and $\eta_{(\sigma)}$ parameters completely represent the S_f and S_e values, is demonstrated by showing that by using the β and $\eta_{(\sigma)}$ parameters we *always can* reproduce the S_f and S_e values.

And because in the Table A-23 of the Shigly's book, for several steel materials, authors present their S_{ut}, σ_t and b values, then in this paper by using the proposed methodology, their corresponding strength β and $\eta_{(\sigma)}$ parameters, the log-mean μ_x and log-standard deviation (σ_x) values, as well as the 95% and 5% reliability percentiles of their S-N curves are all given in section 6. *The novelty* of the given reliability percentiles is that they *do not represent a confidence interval CL* of the S-N curve, instead they represent *a reliability confidence interval* for the S-N curve. But more importantly notice that because the S-N reliability percentiles are the reliability percentiles of the strength $\eta_{(\sigma)}$ parameter, then because in any Weibull analysis the reliability percentiles of $\eta_{(\sigma)}$ are always determined, then automatically we can use these $\eta_{(\sigma)}$ percentiles as the corresponding S-N percentiles. Consequently, any Weibull strength analysis can be seeing as a representation of the reliability percentiles of the related S-N curve [3, 4]. Additionally, because the reliability of the component depends on the applied stress and on its strength, then in section 5, the Weibull strength parameters that represents the desired S-N reliability percentiles, and the Weibull parameters that represents the applied stress, are both used in the stress/strength methodology [5] to determine the reliability of the designed element.

The structure of the paper is as follows. Section 2 presents the generalities of a tensile test. In section 3, the steps of the proposed Weibull/Tensile/Reliability percentiles methodology are given. In section 4, a step-by-step application of the proposed method is given. In section 5, the stress/strength analysis to determine the reliability of the component is presented. In section 6 the Weibull β and $\eta_{(\sigma)}$ parameters, the 95% and 5% reliability percentiles and the corresponding log-mean and log-standard deviation for each one of the steel materials given in the Table A-23 of the Shigly's book are provided. Finally, in section 7, the conclusions are presented.

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2. Tensile Test Generalities

In general, in a tensile test the material properties are directly measured from a sample that is tested at controlled tension force (*F*) until failure. The most general material's properties [2] are the ultimate tensile strength S_{ut}, (it is a measure of the maximum stress that an object/material/structure can withstand without being elongated, stretched or pulled), the true stress σ_t, (it measures the change in the area with respect to time while the specimen is loaded), the maximum elongation (L), and the reduction in the initial area (A_0).

Since these material's properties are random variables, then in the analysis a probability density function (*pdf*) must be used [6] pg.10. In the analysis, the most used *pdfs* are the normal, lognormal and Weibull distributions. Fortunately, as demonstrated in [7], for mechanical stress the best distribution is the Weibull distribution, and from [1] we have that from the Weibull analysis we always can reproduce the analyzed principal stresses (or strength) values. Therefore, in this paper the Weibull distribution is used. Also notice that for $\beta\approx3.4$ the Weibull distribution efficiently mimics the normal distribution, and for $\beta>5$ [8], it efficiently mimics the lognormal distribution.

However, before showing the Weibull distribution completely reproduce the used material's strength values, let first present the generalities of a tensile test formulation.

2.1 General Tensile Test Formulation

In a tensile test analysis, by defining the engineering stress value as $\sigma = F/A_0$, and the engineering strain value as $\varepsilon = \frac{\Delta L}{L_0} = \frac{L-L_0}{L_0}$ where F is the applied force, A_0 is the initial area of the tested element, and L_0 is the initial length, and L is the final elongation of the tested element (see Fig.1).

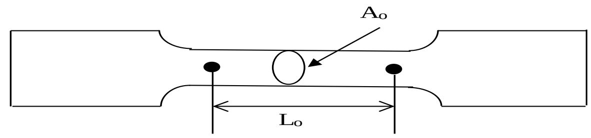

Figure 1. Test Specimen. Source: The Authors

The relationships among the ultimate material's strength S_{ut}, the true stress σ_t, and the true strain ε_t values (see Fig. 2) on which the proposed method is based, are as follows. Based on both F and A_0, the S_{ut} value is defined as

$$S_{ut} = \frac{F}{A_0} \tag{1}$$

Therefore, based on the S_{ut} and ε values the true stress value defined as the instantaneous applied stress, at the S_{ut} coordinate, in terms of the S_{ut} and ε values are determined as

$$\sigma_t = S_{ut}(1-\varepsilon) \tag{2}$$

And the true strain value at the S_{ut} coordinate is given as

$$\varepsilon_t = \ln(1+\varepsilon) \tag{3}$$

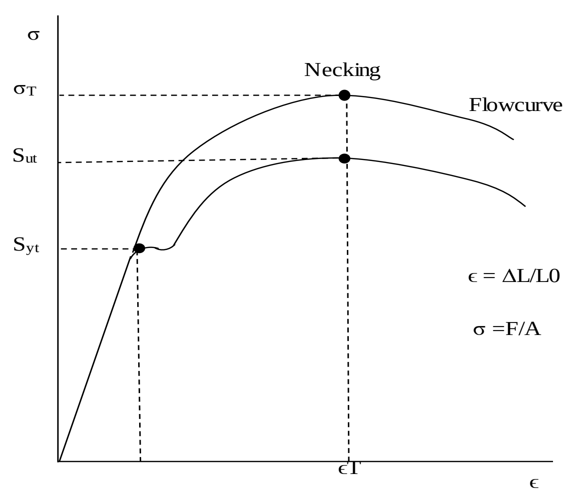

Figure 2. Stress-Strain representation. Source: The Authors

Thus, since now from Eq. (1) the S_{ut} value can be determined, and from Eq. (2), the corresponding σ_t value is given, then now let present how the b value is determined.

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2.2 Fatigue Slope Formulation

In the analysis, the fatigue slope b value of the S-N curve is the exponent that let us to determine the strength range that corresponds to a desired pair of life cycles values [1]. The common approach in the S-N analysis consists in determining b in the logarithm range given by $N_1 = 10^3$ and $N_2 = 10^6$ cycles (see Fig.3). In this logarithm scale the cycles-strength coordinates to determine b are [log $(10^3), \log(fS_{ut})$] and [$\log(10^6), \log(S_e)$]. Where f represents the strength's percentage that the material presents after 10^3 cycles, and S_e represents the corresponding fatigue strength limit.

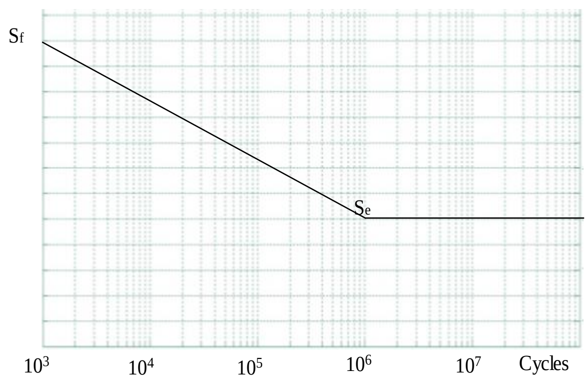

Figure 3. S-N curve representation. Source: The Authors

Hence, since in this logarithm range the S-N curve behavior is linear given as

$$Y_i = a + bX_i \text{ for } i=1,2 \tag{4}$$

Where $Y_1 = log\ (fS_{ut})$, $Y_2 = log\ (S_e)$, $X_1 = log\ (10^3)$ and $X_2 = log\ (10^6)$, then the fatigue b and *parameters of the S-N curve* are determined as

$$b = -\frac{1}{3} log\left(\frac{fS_{ut}}{S_e}\right) \tag{5a}$$

$$a = log\left(\frac{(fS_{ut})^2}{S_e}\right) \tag{5b}$$

Therefore, based on Eqs. (5a and 5b) the relation between the applied stress and its corresponding cycles to failure is given by the Basquin formula given as

$$N_i = \left(\frac{\sigma_{eq}}{a}\right)^{1/b} \tag{5c}$$

However, when S_e is unknown, then the fatigue b value defined in Eq.(5a), based on the σ_t value is given as

$$b = \frac{\log\ (fS_{ut}/\sigma_t)}{\log\ (2N)} \tag{6a}$$

Consequently, the cycles to failure defined in Eq.(5c) based on the σ_t value is given as

$$N_i = \frac{1}{2} log\left(\frac{fS_{ut}}{\sigma_t}\right)^{1/b} \tag{6b}$$

Now that from Eq. (5a and 6a) we can determine the b value, let present the methodology to determine the strength Weibull β and $\eta_{(\sigma)}$ parameters directly from the S_f and S_e values.

3. Weibull/Tensile Test/Reliability Methodology

This section is structured to present *1)* the steps to determine the strength Weibull β and $\eta_{(\sigma)}$ parameters directly from the maximum $S_f = (fS_{ut}) = S_{max}$ and the minimum $(S_e) = S_{min}$ tensile strength values. *2)* how to use the derived β and $\eta_{(\sigma)}$ parameters to determine the reliability percentile of the related S-N curve. And *3)* how to determine the log-standard deviation σ_x value directly from the β value. Let start given the Weibull's generalities.

3.1 Generalities of the Weibull distribution

For the two parameter Weibull distribution [9] given by

$$f(t_i) = \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} exp\left\{-\left(\frac{t_i}{\eta}\right)^{\beta}\right\} \tag{7}$$

Where t represents the desired life time, β is the shape parameter and η is the scale parameter. However, since in this paper the life of the element

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is represented by either its cycles to failure *N*, or by its material's strength σ value, then by replacing *t* in Eq. (7) with either N_i or σ_i, the corresponding Weibull reliability function is given as

$$R(N_i\ or\ \sigma_i) = exp\left\{-\left(\frac{N_i}{\eta_{(N)}}\right)^{\beta}\right\} = exp\left\{-\left(\frac{\sigma_i}{\eta_{(\sigma)}}\right)^{\beta}\right\} \quad (8)$$

From Eq. (8), notice that *1)* although to determine the reliability of the element we can use either N_i or σ_i, the corresponding $\eta_{(N)}$ and $\eta_{(\sigma)}$ values are different ($\eta_{(N)} \neq \eta_{(\sigma)}$). And *2)* the $\eta_{(N)}$ and $\eta_{(\sigma)}$ values are related by the life/stress model, as can be the Arrhenius, the inverse power law model and the Basquin equation defined here in Eq.(5c). Also notice that because in Weibull analysis, by supposing the failure mode remains constant, then in the analysis the β value is considered to be constant [10]. Consequently, as shown in Eq. (8), in any Weibull analysis, we always have two Weibull families. One representing the cycles to failure W(β, $\eta_{(N)}$), and the other representing the material strength W(β, $\eta_{(\sigma)}$). Here the analysis is performed based on the W(β, $\eta_{(\sigma)}$) family. Now let present the steps to determine the β and $\eta_{(\sigma)}$ parameters directly form the tensile $S_f = (fS_{ut}) = S_{max}$ and $(S_e) = S_{min}$ values.

3.2 Steps to Determine the Weibull Strength Parameters

Step1. From the used material determine the corresponding S_{ut}, σ_t and fatigue slope *b* values.

Step2. Determine the desired reliability *R(n)* index to perform the analysis. In practice, it is *R(n)*=0.9535. And it corresponds to test a set of *n*=21 parts [11]. From [11], the relation between *R(n)* and *n* is given as

$$R(n) = \exp\left\{\frac{-1}{n}\right\} \quad (9)$$

Note 1. Here observe *R(n)* is not the reliability of the element, instead *R(n)* is just the reliability on which the analysis will be performed. *R(n)* is alike the confidence interval *CL* used in the quality field.

Step3. By using the *n* value of step 2 in Eq. (10), compute the Y_i elements [12] and its corresponding arithmetic mean μ_y and standard deviation σ_y values as

$$Y_i = ln(-(ln\ (1-(i-0.3)/(n+0.4)))) \quad (10)$$

Note 2. Observe, once *n* was selected in step 2, the μ_y and σ_y values computed from the Y_i elements defined in Eq. (10) are both constant. For *n*=21 (or *R(n)*=0.9535) they are $\mu_y = -0.54562412$ and $\sigma_y = 1.17511694$. In this paper these two values are used.

Step 4. Based on Eq.(6b), by using $N_1 = 10^3$ and the σ_t and *b* values of step1, determine the maximum strength S_f value as

$$S_f = \sigma_t(2N_1)^b \quad (11)$$

Note 3. Observe that because $S_f = f * S_{ut}$, then from Eq. (11) the *f* value is directly given as $f = (S_f)/S_{ut}$.

Step 5. If the S_e value is unknown, then based on Eq.(6b), by using $N_2 = 10^6$ and the σ_t and *b* values of step1 determine the minimum strength S_e value as

$$S_e = \sigma_t(2N_2)^b \quad (12)$$

Step 6. By using the μ_y value from step 3, and the S_f and S_e values, determine the strength Weibull shape parameters as

$$\beta = \frac{-4*\mu_y}{0.99*\ln(S_f/S_e)} \quad (13)$$

Step 7. By using the addressed S_f and S_e values, determine the Weibull scale parameters as

$$\eta_{(\sigma)} = \sqrt[2]{S_f * S_e} \quad (14)$$

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The β and $\eta_{(\sigma)}$ parameters determined in steps 6 and 7 are the parameters of the Weibull strength distribution.

Note 4. Notice if f, S_{ut}, and S_e are known then from Eq.(5a) *b* can be estimated, implying the true stress σ_t value is not necessary. It is to say, as shown in Eqs. (13 and 14), the Weibull strength parameters only depends on the S_f and S_e values.

Now based on the β and $\eta_{(\sigma)}$ parameters let determine the corresponding log-mean μ_x and log-standard σ_x deviation values used to formulate the confidence interval of μ_x.

3.3 Steps to Determine the Log-mean and the Log-standard Deviation

The analysis is based on the linear form of the reliability function [2] defined in Eq.(9) given as

$$Y_i = b_0 + \beta X_i \tag{15}$$

Thus, since from Eq. (15) $X_i = \ln(t_i)$, then we need to determine its log-mean μ_x and its log-standard deviation σ_x values. From [1] the μ_x value is directly given by the strength scale $\eta_{(\sigma)}$ parameters as

$$\mu_x = \ln(\eta_{(\sigma)}) \tag{16a}$$

And from [13], based on both the μ_y value of step 3, and on the addressed β value, the σ_x value is given as

$$\sigma_x = \frac{\sigma_y}{\beta} \tag{16b}$$

Thus, a confidence interval (*CL*) of μ_x is given as

$$CL = \mu_x \pm Z_{\alpha/2}\sigma_x \tag{17}$$

Where $Z_{\alpha/2}$ is the *th* desired percentile given by the normal distribution, (which for *CL*=0.95, is $Z_{0.1/2} = 1.644853$).

Unfortunately, although from Eq. (16a) $\mu_x = \ln(\eta_\sigma)$, the *CL* limits defined in Eq. (17) cannot be used to determine a confidence interval for $\eta_{(\sigma)}$.

Consequently, Eq. (17) cannot be used to determine the reliability percentiles of the S-N curve neither. This fact occurs because there is not a direct relationship between *CL* and *R(t)*. *CL* represents an instantaneous probability that the strength of *n* identical components behaves around μ_x, and *R(t)* represents the probability that a observed (measured) μ_x value stay around this value through the time. It is to say, while the *CL* value depends only on the lack of homogeny of the material, the *R(t)* index depends also on the applied stress, the desired time *t*, and on the observed μ_x value. Thus, Eq. (17) should not be used to determine the S-N percentiles that represents the desired *R(t)* index. Numerically, the deficiency of using *CL* in reliability analysis is given in section 4.2.

Here notice that in contrast to Eq. (17), in reliability analysis we are interested *only in the upper limit*. Consequently, since from Eq. (8) the *R(t)* index depends only on the $\eta_{(\sigma)}$ value, then because $\mu_x = \ln(\eta_{(\sigma)})$, in the analysis μ_x is the lower allowed value that we can used to design the element. Therefore, as shown in [14] if $\mu_x = \ln(\eta_{(\sigma)})$ is going to be monitored in a process, then in the monitoring control chart the μ_x value must be set us the lower allowed value.

Now based on the addressed μ_x and σ_x values, let present the formulation to determine the reliability percentile of the related S-N curve.

3.4 Reliability Percentiles for the S-N Curve

The efficiency of the proposed method is based on the following two facts.

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1) Since from Eq.(14), $\eta_{(\sigma)}$ is given as the square root of the product of S_f, and S_e, then in logarithm scale $\mu_x = \ln(\eta_{(\sigma)})$ is the average between S_f and S_e, implying that $\ln(S_f) - \ln(\eta_\sigma) = \ln(\eta_\sigma) - \ln(S_e)$ or equivalently that the relation given in Eq.(18) always holds

$$\ln(S_f/\eta_\sigma) = \ln(\eta_\sigma/S_e) \qquad (18)$$

2) Because in logarithm scale the three values, $\ln(S_f)$, $\ln(\eta_\sigma)$ and $\ln(S_e)$, all are in the same S-N line, then this line represents the lower th-reliability percentile for which it is expected the product present the desired *R(t)* index. Consequently, from Eq. (18) and Eq. (8), we have that the following reliability relationship always holds

$$R(\sigma) = exp\left\{-\left(\frac{\eta_{(\sigma)}}{\eta_{\sigma Ni}}\right)^\beta\right\} = exp\left\{-\left(\frac{S_{fi}}{S_f}\right)^\beta\right\} = exp\left\{-\left(\frac{S_{ei}}{S_e}\right)^\beta\right\} \qquad (19)$$

Eq. (19) implies that in practice, *the derived reliability percentiles of the S-N curve can also be used as the minimum strength $\eta_{(\sigma i)}$ value that the used material must present to have the desired reliability*. Now based on the above two facts, the steps to determine the reliability percentiles of the S-N curve are as follows.

3.4.1 Steps to Determine the Reliability Percentiles for the S-N Curve

Step 1. Determine the Y_i element that corresponds to the desired upper reliability percentile of the S-N curve as

$$Y_{ui} = ln(-(ln\,(R(t_{ui}))) \qquad (20a)$$

Step 2. Determine the Y_i element that corresponds to the desired lower reliability percentile of the S-N curve as

$$Y_{Li} = ln(-(ln\,(1 - R(t_{Li}))) \qquad (20b)$$

Step 3. By using the Y_{ui} value of step1, determine the upper values of S_f, η_σ, and S_e that corresponds to the upper reliability percentile of the S-N curve as

$$S_{fu} = \frac{S_f}{Exp\{Y_{ui}/\beta\}}\,;\, \eta_{(\sigma u)} = \frac{\eta_\sigma}{Exp\{Y_{ui}/\beta\}}\,;\, S_{eu} = \frac{S_e}{Exp\{Y_{ui}/\beta\}} \qquad (21)$$

Step 4. By using the Y_{Li} value of step 2, determine the lower value of S_f, $\eta_{(\sigma)}$, and S_e that corresponds to the lower reliability percentile of the S-N curve as

$$S_{fL} = \frac{S_f}{Exp\{Y_{Li}/\beta\}}\,;\, \eta_{(\sigma L)} = \frac{\eta_{(\sigma)}}{Exp\{Y_{Li}/\beta\}}\,;\, S_{eL} = \frac{S_e}{Exp\{Y_{Li}/\beta\}} \qquad (22)$$

Step 5. Plot the upper and lower reliability percentiles.

Now let present the numerical application.

4. Numerical Application

As an application let used data given in the first row of Table A-23 of the Shigly's book. The material is the steel grade (a) A538A (b). For this material, the Weibull strength parameters of section 3.2 are as follows.

4.1 Weibull Strength Parameters

Step 1. The corresponding strength data are $S_{ut} = 1515MPa$, $\sigma_t = 1655MPa$ and fatigue slope *b*=–0.065.
Step 2. Suppose *R(n)*=0.9535 is desired.
Step 3. The Y_i elements are given in Table 1. From these data $\mu_y = -0.54562412$ and $\sigma_y = 1.17511694$.
Step 4. The maximum strength is $S_f = 1655(2 * 1000)^{-0.065} = 1009.79MPa$.
Step 5. The minimum strength is $S_e = 1655(2 * 1000{,}000)^{-0.065} = 644.51MPa$.
Step 6. The Weibull shape parameter is $\beta = \frac{-4*(-0.54562412)}{0.99*\ln(1009.79/644.51)} = 4.909848$.

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Step 7. The Weibull scale parameter is $\eta_{(\sigma)} = \sqrt[2]{1009.79 * 644.51} = 806.7353MPa.$

Therefore the Weibull strength distribution to the steel grade (a) A538A (b) material is W(4.909848, 806.7353MPa).

Now based on these parameters let determine the corresponding log-mean μ_x and log-standard deviation σ_x values mentioned in section 3.3.

Table 1. Elements of vector Y by using Eq.(10)

n	1	2	3	4	5	6	7	8	9	10	11
Y_i	-3.403483	-2.491662	-2.003463	-1.6616459	-1.3943983	-1.1720537	-0.9793812	-0.807447	-0.6504921	-0.50450882	-0.366512921
n	12	13	14	15	16	17	18	19	20	21	μy=-0.54562412
Y_i	-0.234122	-0.105285	0.0219284	0.1495258	0.279845	0.4159621	0.56250196	0.7276158	0.92931067	1.22965981	σy=1.17511694

Source: The Authors

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4.2 Log-mean and Log-standard Deviation

From Eq. (16a), the log-mean is $\mu_x = \ln(806.7353) = 6.692995$ and from Eq.(16b) the log-standard deviation is $\sigma_x = \frac{1.17511694}{4.909848} = 0.239338$, (observe both μ_x and σ_x were determined without any observed failure time data). Therefore, from Eq.(17), the 95% confidence interval for μ_x is $CL = 6.692995 \pm 1.644853 * 0.239338$; $[6.299319 \leq \mu_x \leq 7.086673]$ or equivalently because from Eq.(16a) $\mu_x = \ln(\eta_{(\sigma)})$, then by taking the exponential, the 95% confidence interval for η_σ is $[544.2009MPa \leq \eta_{(\sigma)} \leq 1195.9219MPa]$, unfortunately as shown next, this confidence interval should not be used in reliability analysis. For example, notice that although under probabilistic point of view we can say with a confidence level of 95% the lower expected value of the Weibull scale parameter is $\eta_{(\sigma L)} = 544.2009MPa$, and then it should be monitored in the production process in logarithm scale as in Fig.4 and/or in natural scale as in Fig.5

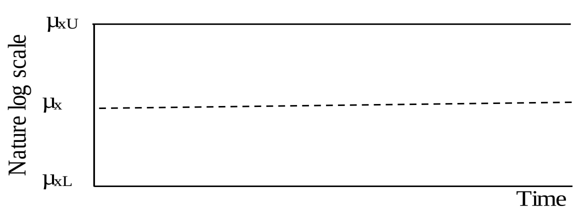

Figure 4. Control Chart for μx (logarithm Scale). Source: The Authors

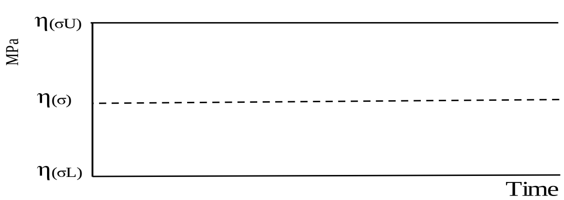

Figure 5. Control Chart for the Weibull scale parameter. Source: The Authors

Unfortunately, as mentioned above in reliability, monitoring (or using) the lower limit of $\eta_{(\sigma)}$ *is not correct* because in reliability the addressed $\eta_{(\sigma)}$ value (or nominal μ_x value) is the lower allowed value. Thus, in the monitoring process, the $\eta_{(\sigma)}$ value (or equivalently the μ_x value) is the one that must be set as the lower allowed limit in the control chart (see Fig.6 and Fig.7).

Figure 6. Control Chart for μx (logarithm Scale). Source: The Authors

Figure 7. Control Chart for the Weibull scale parameter. Source: The Authors

Additionally, it is shown that although by using the *CL* limits defined in Eq. (17), the 95% confidence for the S-N curve plotted in Fig.8 is possible, they do not the 95% reliability confidence interval for the S-N curve. Consequently, because the *CL* confidence interval is not a reliability percentile, then by using the *CL* values in Eq. (19), the estimated reliability is not the desired *R(t)*=0.95 index.

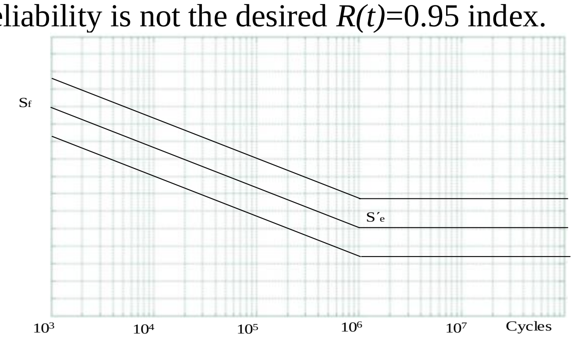

Figure 8. Probabilistic Percentiles for the S-N curve. Source: The Authors

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Seeing this observe that by using the upper and lower limits of *CL* to determine *R(σ)*, the demonstrated reliability is not the desired one. For the upper level $\eta_{(\sigma U)} = 1195.9219MPa$, then with $\eta_{(\sigma)} = 806.7356MPa$ in Eq.(19), the estimated reliability instead of be $R(\sigma) = 0.95$ is only $R(\sigma_U) = exp\left\{-\left(\frac{806.7356}{1195.9219}\right)^{4.909848}\right\} = 0.8653$.

Similarly, if we use the lower confidence level $\eta_{(\sigma U)} = 544.2009MPa$ with $\eta_{(\sigma)} = 806.7356MPa$ in Eq.(19), the estimated reliability index instead of be $R(\sigma) = 0.95$, also is only of $R(\sigma_L) = exp\left\{-\left(\frac{544.2009}{806.7356}\right)^{4.909848}\right\} = 0.8653$.

Therefore, the general conclusion is that by using the *CL* limits in reliability analysis we sub-estimate the real *R(σ)* index (0.8653<0.95) of the element, and consequently the *CL* limits should not be used in the reliability analysis.

Now we know the *CL* values should not be used, let determine the reliability percentiles for the S-N curve that we can use in any reliability analysis. Following section 3.4.1, the analysis is as follows.

4.3 Reliability Percentiles for the S-N Curve

The reliability percentile analysis for the S-N curve is as follows

Step 1. From Eq.(20a) the upper Y_i element for *R(t)*=0.95 is $Y_{ui} = ln(-(ln\,(0.95))) = -2.970195249$.

Step 2. From Eq.(20b) the lower Y_i element for *R(t)*=0.05 is $Y_{Li} = ln(-(ln\,(1-0.95))) = 1.0971887$.

Step 3. From Eq. (21) the upper strength values are

$$S_{fu} = \frac{1009.79MPa}{Exp\{-2.970195249/4.909848\}} = 1849.08MPa.$$

$$\eta_{(\sigma u)} = \frac{806.7353MPa}{Exp\{-2.970195249/4.909848\}} = 1477.26MPa$$

and $S_{eu} = \frac{644.51MPa}{Exp\{-2.970195249/4.909848\}} = 1180.20MPa$.

Step 4. From Eq. (22) the lower strength values are

$$S_{fL} = \frac{1009.79MPa}{Exp\{1.0971887/4.909848\}} = 807.57MPa,$$

$$\eta_{(\sigma L)} = \frac{806.7353MPa}{Exp\{1.0971887/4.909848\}} = 645.18MPa \text{ and}$$

$$S_{eL} = \frac{644.51MPa}{Exp\{1.0971887/4.909848\}} = 515.44MPa.$$

From the above data, notice because the Y_{ui} value was determined by using $R(\sigma) = 0.95$, then by using the S_{fu}, $\eta_{(\sigma u)}$ and S_{eu} values in Eq. (19), the reliability percentile is always $R(\sigma) = 0.95$.

For $R(\sigma/S_f, S_{fu}) = exp\left\{-\left(\frac{1009.79}{1849.08}\right)^{4.909848}\right\} = 0.95$, $R(\sigma/\eta_{(\sigma)}, \eta_{(\sigma u)}) = exp\left\{-\left(\frac{806.7356}{1477.26}\right)^{4.909848}\right\} = 0.95$, and $R(\sigma/S_e, S_{eu}) = exp\left\{-\left(\frac{644.51}{1180.20}\right)^{4.909848}\right\} = 0.95$.

Similarly, since the Y_{Li} value was determined by using $R(\sigma) = 0.05$, then by using the S_{fL}, $\eta_{(\sigma L)}$ and S_{eL} values in Eq. (19), the reliability percentile in all cases is always $R(\sigma) = 0.05$.

For $R(\sigma/S_f, S_{fL}) = exp\left\{-\left(\frac{1009.79}{807.57}\right)^{4.909848}\right\} = 0.05$, $R(\sigma/\eta_{(\sigma)}, \eta_{(\sigma L)}) =$

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$exp\left\{-\left(\frac{806.7356}{645.18}\right)^{4.909848}\right\} = 0.05$ and $R(\sigma/S_e, S_{eL}) = exp\left\{-\left(\frac{644.51}{515.44}\right)^{4.909848}\right\} = 0.05.$

The corresponding percentiles of the S-N curve in MPa and in logarithm scale are all given in Table 2.

Table 2. Reliability Percentiles for the S-N curve of the A5

Percentiles in Mpa Values				Percentile
Limits	Sf	η(σ)	Se	ln(Sf)
Upper	1849.08	1477.26	1180.20	7.5224
Mean	1009.79	806.74	644.51	6.9175
Lower	807.57	645.18	515.44	6.6940

Source: The Authors

Here it is very important to notice from either Table 2 or Figure 9 that data in MPa do not fall in a right line with the $\eta_{(\sigma)}$ value.

In contrast observe from Fig. 10 that in logarithm scale they are in line with the $\eta_{(\sigma)}$ value. Also notice from Fig.9 and Fig.10 that the upper and lower percentiles are not symmetric around the $\eta_{(\sigma)}$ value, and that this fact is due to in Weibull analysis, the $\eta_{(\sigma)}$ does not represent the 0.50 percentile, instead it represents the 0.6321 failure percentile, implying the limits around the $\eta_{(\sigma)}$ value never will be symmetric around the $\eta_{(\sigma)}$ value.

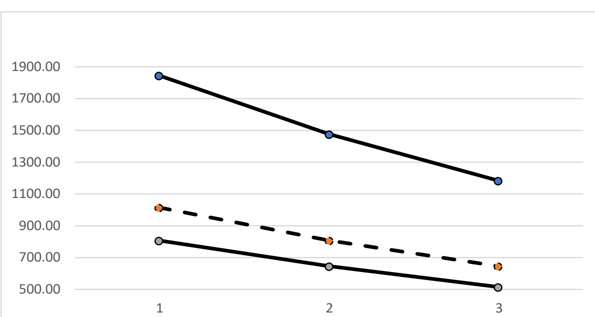

Figure 9. S-N curve in MPa values. Source: The Authors

Additionally, remember that as shown in Eq. (18), the symmetrical behavior around $\eta_{(\sigma)}$ occurs only for the S_f and S_e values from which the $\eta_{(\sigma)}$ value was determined. In order to clarify the mentioned facts, in Table 3 the Weibull analysis for the expected values of $\eta_{(\sigma)}$ are given.

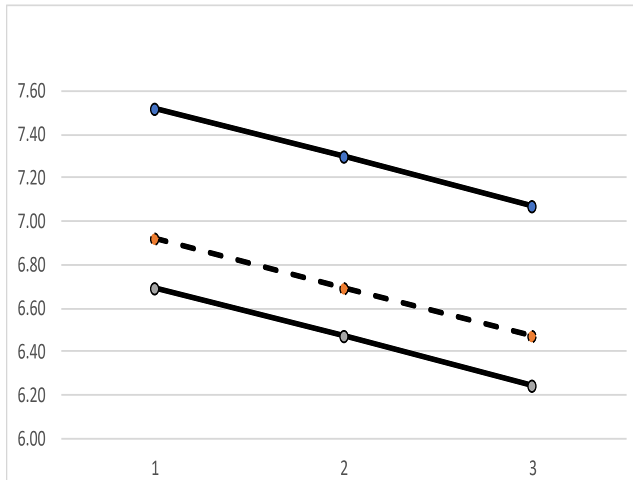

Figure 9. S-N curve in logarithm scale. Source: The Authors

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Table 3. Weibull Scale Analysis

n	Yi	Yui	σi	η(σi)	$R(t)$
1	-3.4035	0.5000	403.35	1613.55	0.9673
	-2.9702	**0.5461**	**440.56**	**1477.26**	**0.9500**
2	-2.4917	0.6020	485.66	1340.07	0.9206
3	-2.0035	0.6649	536.44	1213.23	0.8738
4	-1.6616	0.7129	575.11	1131.64	0.8271
5	-1.3944	0.7528	607.28	1071.69	0.7804
6	-1.1721	0.7876	635.42	1024.24	0.7336
	-1.1023	**0.7989**	**644.51**	**1009.79**	**0.7174**
7	-0.9794	0.8192	660.85	984.83	0.6869
8	-0.8074	0.8484	684.40	950.94	0.6402
9	-0.6505	0.8759	706.63	921.02	0.5935
10	-0.5045	0.9023	727.96	894.04	0.5467
11	-0.3665	0.9281	748.71	869.26	0.5000
12	-0.2341	0.9534	769.17	846.14	0.4533
13	-0.1053	0.9788	789.62	824.22	0.4065
	0.0000	**1.0000**	**806.735**	**806.735**	**0.3679**
14	0.0219	1.0045	810.35	803.14	0.3598
15	0.1495	1.0309	831.68	782.54	0.3131
16	0.2798	1.0587	854.05	762.04	0.2664
17	0.4160	1.0884	878.06	741.20	0.2196
18	0.5625	1.1214	904.66	719.41	0.1729
19	0.7276	1.1597	935.60	695.62	0.1262
20	0.9293	1.2084	974.84	667.62	0.0794
	1.0972	1.2504	1008.74	645.18	0.0500
	1.1023	**1.2517**	**1009.79**	**644.51**	**0.0492**
21	1.2297	1.2846	1036.33	628.00	0.0327

Source: The Authors

The practical interpretation of data given in Table 3 is as follows.

1. The values of the column σ_i in Table 3 represent the maximum applied stress values for which a product that has the $\eta_{(\sigma)}$ strength value, will present the reliability *R(t)* index given in the row of Table 3 that corresponds to the selected σ_i value. For example, if a component (material) with strength of $\eta_{(\sigma)} = 806.7353MPa$, is subjected *to constant stress* of σ =403.35MPa, then as shown in Table 3, it is expected the element will present a minimum reliability of $\exp\left\{-\left(\frac{403.35}{806.7353}\right)^{4.909848}\right\} = 0.9673$. In Table 3, by using the Y_i value defined in Eq. (10), the corresponding σ_i value was determined as

$$\sigma_i = \eta_{(\sigma)} * \exp\{Y_i/\beta\} \quad (23)$$

2. The values of the column $\eta_{(\sigma i)}$ in Table 3, represent the strength value that a product should has to present the given reliability *R(t)* index when the applied stress is constant at the $\eta_{(\sigma)}$ value. For example, the $\eta_{(\sigma i)} = 1613.55MPa$ value given in the first row of Table 3, represents the minimum strength value that a product (material) must have to presents a reliability of $R(t) = 0.9673$ when *the maximum applied stress is constant* at the value of $\eta_{(\sigma)} = 806.7353MPa$. It is to say $R(t) = \exp\left\{-\left(\frac{806.7353}{1613.55}\right)^{4.909848}\right\} = 0.9673$. In Table 3, the $\eta_{(\sigma i)}$ value was determined as

$$\eta_{(\sigma i)} = \eta_{(\sigma)}/\exp\{Y_i/\beta\} \quad (24)$$

From Table 3 also notice the rows where the Weibull analysis reproduce the $S_f = 1009.79MPa$ and $S_e = 644.51MPa$ values, as well as the upper 95% and lower 5% percentiles of $\eta_{(\sigma)}$ were also added. Also from Table 3, notice that as shown in Fig. 9 and in Fig. 10 the behavior around the $\eta_{(\sigma)}$ value is not symmetrical. Now let determine the reliability of a component by using the stress/strength analysis.

5. Stress/Strength Analysis

Since all mechanical element is subjected to an applied stress and it has an inherent strength to overcome the applied stress, then because both the stress and the strength are random variable, the element's reliability must be determined based on the distribution that represent the applied stress, and on the distribution that

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represent the inherent strength. Therefore, the right reliability function to be used in the analysis of a mechanical element is the composed reliability function known as a stress/strength reliability function [15]. In this stress/strength approach any pair of combination of stress and strength functions is possible. However, the most common combinations are the normal/normal, the log-normal/log-normal, the Weibull/Weibull and any pair of combination among these three distributions [16]. But because here the analysis is a stress-based analysis which is efficiently modeled by the Weibull distribution, then the Weibull/Weibull approach is used as follows.

5.1 Numerical Analysis

In this section, *the strength Weibull distribution* data addressed in section 4.1 of the steel grade (a) A538A (b) material is used. From this section the addressed Weibull strength family is W (β=4.909848, $\eta_{(\sigma)}$=806.7353MPa). Therefore, to apply the stress/strength analysis the corresponding stress Weibull distribution must be addressed. Doing this, suppose from an application the maximum principal applied stress is $\sigma_1 = 600MPa$ and the minimum principal applied stress that generates a failure is $\sigma_2 = 380MPa$. (σ_1 and σ_2 are the principal stresses given by the Mohr circle analysis).

Thus, with these two principal stress values, from Eq. (14) the scale Weibull stress parameter is $\eta_s = \sqrt[2]{600 * 380} = 447.4935MPa$, and from Eq. (13) β=4.909848. Thus, the Weibull stress distribution is Ws(β=4.909848, η_s=477.4935MPa). Consequently, from the Weibull/Weibull stress/strength reliability function [1] given as

$$R(t/\eta_s, \eta_{(\sigma i)}) = \frac{\eta_{(\sigma i)}{}^\wedge\beta}{\eta_{(\sigma i)}{}^\wedge\beta + \eta_s{}^\wedge\beta} \qquad (25)$$

Therefore, the reliability of the designed component is

$$R(t, \eta_s, \eta_{(\sigma i)}) = \frac{806.7353^{4.909848}}{806.7353^{4.909848} + 477.4935^{4.909848}}$$
$= 0.9292$.

Finally, it is important to observe because the reliability index given in Table 3 and that given from Eq. (25) tends to be the same for high reliability indices, (say a reliability above 0.90), then the reliability of an element can be determined directly by using the Weibull strength parameters as in Table 3, or by using the stress and strength distributions in Eq. (25).

Seeing this numerically, suppose that in an application the used material is subjected to reversible stress with Weibull stress parameter η_s=403.35MPa. Therefore, from Eq. (25), ***as shown in Table 3***, the estimated reliability is

$$R(t, \eta_s, \eta_{(\sigma i)}) = \frac{806.7353^{4.909848}}{806.7353^{4.909848} + 403.35^{4.909848}}$$

Similarly, if the applied stress is η_s=536.44MPa, then it is $R(t, \eta_s, \eta_{(\sigma i)}) = \frac{806.7353^{4.909848}}{806.7353^{4.909848} + 536.44^{4.909848}} = 0.8811$. For detail of the given formulation see [1].

Consequently, for high reliability indices, the σi column of any ***Weibull Strength*** analysis can be used as the maximum allowed constant stress value that we can apply, in order the component presents the desired reliability. Similarly, the $\eta_{(\sigma i)}$ column of any ***Weibull Strength*** analysis can be used as the minimum allowed strength value that the used material must present, in order the designed element present the desired reliability when it is subjected to a maximum stress value represented by the strength scale $\eta_{(\sigma)}$ value. Now by using the proposed Weibull/S-N methodology, the Weibull parameters, the log-

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mean and log-standard deviation parameters and the 0.95 and 0.05 reliability percentiles of each one of the steel materials given in Table A-23 of the Shigly's book are all given in Table 4.

6. Weibull/S-N analysis for Materials given in Table A-23 of the Shigly's book.

The analysis is presented in Table 4.

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Table 4. Weibull Strength Parameters, Log-Parameters and Reliability Percentiles for Tensile Test Data given in Table A-23 of the Shigly's book

Steel	Ultimate	True	Fatigue	Strength at	Strength at	Weibull Parameters		Log-Parameters		Reliability Percentiles for the S-N Curve					
Grade	Strength	Stress	Exponent	N1=10^3	N2=10^6	Shape	Scale	Mean	Stdev	R(0.95), Yui=-2.970195249			R(0.05), YLi=1.0971887		
	(MPa)	(MPa)	b	Sf	Se	β	η(σ)	μx	σx	Sf	η(σ)	Se	Sf	η(σ)	Se
A538A (b)	1515	1655	-0.065	1009.79	644.51	4.909848	806.7353	6.6930	0.23934	1849.08	1477.26	1180.20	807.57	645.18	515.44
A538B (b)	1860	2135	-0.071	1244.59	762.12	4.494931	973.9233	6.8813	0.26143	2409.91	1885.82	1475.71	975.03	762.98	597.06
A538C (b)	2000	2240	-0.070	1315.76	811.29	4.559144	1033.1798	6.9404	0.25775	2524.12	1982.03	1556.36	1034.33	812.19	637.76
AM-350 (c)	1315	2800	-0.140	966.08	367.29	2.279572	595.6811	6.3897	0.51550	3555.35	2192.21	1351.71	597.01	368.11	226.98
AM-350 (c)	1905	2690	-0.102	1238.93	612.42	3.128824	871.0582	6.7697	0.37558	3201.28	2250.73	1582.43	872.47	613.41	431.27
Gainex (c)	530	805	-0.070	472.85	291.56	4.559144	371.2990	5.9170	0.25775	907.11	712.29	559.32	371.71	291.88	229.20
Gainex (c)	510	805	-0.071	469.27	287.36	4.494931	367.2170	5.9060	0.26143	908.66	711.05	556.42	367.63	287.68	225.12
H-11	2585	3170	-0.077	1765.55	1037.24	4.144676	1353.2559	7.2103	0.28352	3615.00	2770.82	2123.77	1354.92	1038.51	796.00
RQC-100 (c)	940	1240	-0.070	728.37	449.11	4.559144	571.9388	6.3490	0.25775	1397.28	1097.20	861.56	572.58	449.61	353.05
RQC-100 (c)	930	1240	-0.070	728.37	449.11	4.559144	571.9388	6.3490	0.25775	1397.28	1097.20	861.56	572.58	449.61	353.05
10B62	1640	1780	-0.067	1069.67	673.37	4.763285	848.6937	6.7437	0.24670	1995.54	1583.29	1256.20	849.60	674.08	534.83
1005-1009	360	580	-0.090	292.64	157.16	3.546001	214.4546	5.3681	0.33139	676.25	495.57	363.17	214.76	157.38	115.33
1005-1009	470	515	-0.059	328.89	218.80	5.409154	268.2541	5.5919	0.21725	569.53	464.54	378.90	268.51	219.01	178.63
1005-1009	415	540	-0.073	310.04	187.25	4.371782	240.9454	5.4846	0.26880	611.62	475.31	369.38	241.23	187.47	145.69
1005-1009	345	640	-0.109	279.49	131.63	2.927891	191.8084	5.2565	0.40135	770.79	528.98	363.03	192.14	131.86	90.49
1015	415	825	-0.110	357.55	167.24	2.901273	244.5348	5.4994	0.40503	995.30	680.69	465.53	244.96	167.53	114.58
1020	440	895	-0.120	359.50	156.93	2.659501	237.5196	5.4703	0.44186	1098.32	725.65	479.44	237.97	157.23	103.88
1040	620	1540	-0.140	531.34	202.01	2.279572	327.6246	5.7919	0.51550	1955.45	1205.72	743.44	328.36	202.46	124.84
1045	725	1225	-0.095	595.03	308.70	3.359369	428.5862	6.0605	0.34980	1440.53	1037.58	747.34	429.24	309.17	222.69
1045	1450	1860	-0.073	1067.92	644.97	4.371782	829.9229	6.7213	0.26880	2106.68	1637.19	1272.32	830.89	645.72	501.81
1045	1345	1585	-0.074	903.14	541.69	4.312704	699.4441	6.5503	0.27248	1798.27	1392.69	1078.59	700.27	542.33	420.01
1045	1585	1795	-0.070	1054.37	650.12	4.559144	827.9276	6.7189	0.25775	2022.68	1588.28	1247.17	828.85	650.84	511.07
1045	1825	2275	-0.080	1238.51	712.69	3.989251	939.5048	6.8454	0.29457	2607.67	1978.12	1500.56	940.70	713.60	541.32
1045	2240	2275	-0.081	1229.13	702.42	3.940001	929.1759	6.8343	0.29825	2612.12	1974.67	1492.77	930.38	703.33	531.69
1144	930	1000	-0.080	544.40	313.27	3.989251	412.9691	6.0234	0.29457	1146.23	869.50	659.59	413.50	313.67	237.94
1144	1035	1585	-0.090	799.72	429.47	3.546001	586.0525	6.3734	0.33139	1848.03	1354.28	992.45	586.89	430.09	315.18
1541F	950	1275	-0.076	715.54	423.28	4.199212	550.3410	6.3105	0.27984	1451.50	1116.40	858.65	551.01	423.80	325.95
1541F	890	1275	-0.071	743.25	455.13	4.494931	581.6169	6.3658	0.26143	1439.17	1126.19	881.28	582.27	455.65	356.56
4130	895	1275	-0.083	678.46	382.41	3.845061	509.3598	6.2332	0.30562	1468.94	1102.82	827.95	510.03	382.91	287.47
4130	1425	1695	-0.081	915.77	523.34	3.940001	692.2871	6.5400	0.29825	1946.18	1471.23	1112.20	693.18	524.02	396.14
4140	1075	1825	-0.080	993.53	571.72	3.989251	753.6687	6.6250	0.29457	2091.87	1586.84	1203.74	754.63	572.44	434.24
4142	1060	1450	-0.100	678.06	339.83	3.191401	480.0262	6.1738	0.36821	1719.72	1217.47	861.90	480.79	340.37	240.97
4142	1250	1250	-0.080	680.50	391.59	3.989251	516.2114	6.2465	0.29457	1432.79	1086.88	824.48	516.87	392.09	297.43
4142	1415	1825	-0.080	993.53	571.72	3.989251	753.6687	6.6250	0.29457	2091.87	1586.84	1203.74	754.63	572.44	434.24
4142	1550	1895	-0.090	956.13	513.47	3.546001	700.6748	6.5520	0.33139	2209.48	1619.16	1186.56	701.68	514.21	376.82
4142	1760	2000	-0.080	1088.80	626.54	3.989251	825.9382	6.7165	0.29457	2292.46	1739.01	1319.17	826.99	627.34	475.88
4142	2035	2070	-0.082	1109.91	629.92	3.891952	836.1532	6.7288	0.30194	2380.80	1793.59	1351.21	837.25	630.74	475.17
4142	1930	2105	-0.090	1062.09	570.37	3.546001	778.3221	6.6571	0.33139	2454.33	1798.59	1318.05	779.44	571.19	418.58
4142	1930	2170	-0.081	1172.40	670.00	3.940001	886.2909	6.7870	0.29825	2491.56	1883.53	1423.88	887.43	670.87	507.15
4142	2240	1655	-0.089	841.41	454.99	3.585844	618.7373	6.4277	0.32771	1926.36	1416.57	1041.69	619.61	455.64	335.06
4340	825	1200	-0.095	582.89	302.40	3.359369	419.8396	6.0399	0.34980	1411.13	1016.40	732.09	420.48	302.86	218.14
4340	1470	2000	-0.091	1001.47	534.12	3.507034	731.3685	6.5949	0.33507	2335.88	1705.89	1245.81	732.43	534.89	390.63
4340	1240	1655	-0.076	928.79	549.44	4.199212	714.3642	6.5714	0.27984	1884.11	1449.12	1114.57	715.23	550.10	423.10
5160	1670	1930	-0.071	1125.08	688.94	4.494931	880.4084	6.7804	0.26143	2178.52	1704.75	1334.01	881.40	689.72	539.73
52100	2015	2585	-0.090	1304.27	700.43	3.546001	955.8018	6.8626	0.33139	3013.98	2208.72	1618.60	957.17	701.44	514.03
9262	925	1040	-0.071	606.26	371.24	4.494931	474.4169	6.1621	0.26143	1173.92	918.62	718.85	474.95	371.66	290.84
9262	1000	1220	-0.073	700.46	423.04	4.371782	544.3580	6.2996	0.26880	1381.80	1073.85	834.54	544.99	423.54	329.15
9262	565	1855	-0.057	1202.78	811.31	5.598949	987.8368	6.8955	0.20988	2044.44	1679.09	1379.03	988.73	812.04	666.93
9050C (d)	565	1170	-0.120	469.96	205.15	2.659501	310.5005	5.7382	0.44186	1435.80	948.62	626.75	311.09	205.54	135.80
9050C (d)	565	970	-0.110	420.40	196.63	2.901273	287.5136	5.6613	0.40503	1170.23	800.33	547.36	288.02	196.98	134.72
9050X (d)	440	625	-0.075	353.43	210.52	4.255201	272.7739	5.6086	0.27616	710.31	548.21	423.10	273.10	210.78	162.68
9050X (d)	530	1005	-0.100	469.96	235.54	3.191401	332.7078	5.8073	0.36821	1191.94	843.83	597.39	333.24	235.91	167.01
9050X (d)	695	1055	-0.08	574.34	330.50	3.989251	435.6824	6.0769	0.29457	1209.27	917.33	695.86	436.24	330.92	251.03

7. Conclusions

1. Although the relation $\mu_x = \ln(\eta_{(\sigma)})$ holds, the confidence interval *CL* limits of a S-N curve defined in Eq. (17), should not be used to perform a reliability analysis. They sub-estimate the reliability index.

2. From Eqs. (21 and 22) the upper and lower S_f, $\eta_{(\sigma)}$, and S_e values to determine any desired reliability percentile for a S-N curve are given by using only the corresponding $Y_{L,ui}$ and β values.
3. Observe that although here the Weibull strength parameters were both determined for $N_1 = 10^3$ and $N_2 = 10^6$, any other desired values between these two values can be used.

4. As shown in Table 3, the lower reliability percentiles of the S-N curve are the minimum strength values given in the column $\eta_{(\sigma i)}$ of Table

5. Due to the column $\eta_{(\sigma i)}$ of Table 3 represents the minimum strength values that the designed element must have to present the desired reliability, then the reliability percentiles of the S-N curve can be used as the accelerated levels in and ALT test to demonstrate the product presents the intended reliability [17].

6. Although the Weibull analysis performed in Table 3 is for constant stress values, and that given by the stress/strength methodology is for variable stress behavior, for high reliability $= 0.9678.$ indexes, the estimated reliability indexes are both similar [18] $[R(\sigma) \cong R(t, \eta_s, \eta_{(\sigma i)})]$. Formal formulation why this fact occurs is an open issue on which more research must be undertaken.

8. Authorship acknowledgements

Manuel Baro Tijerina: Conceptualización; Ideas; Metodología; Análisis formal; Investigación; Borrador original. *Manuel R. Piña Monarrez:* Conceptualización; Ideas; Escritura. *Alberto Jesús Barraza Contreras:* Análisis de datos; Escritura; Borrador original; Revisión y edición.

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